Mike R. Jeffrey*

Modeling with nonsmooth dynamics

January 10, 2020

Springer

*Department of Engineering Mathematics, University of Bristol, Merchant Venturer's Building, Bristol BS8 1UB, UK

Contents

Preface

As mathematics is applied to model ever new problems in engineering and the life sciences, increasing use is being made of systems that switch between different sets of equations on distinct domains. To find their dynamics requires the discontinuity between domains to be resolved or 'regularized' in some way, and there exist a range of methods to do so. Some preserve the ideal character of the discontinuity as a piecewise-smooth system (giving e.g. 'impact' or 'switching' dynamics), while others blur the discontinuity by smoothing it out, or introducing overshoots due to deterministic or stochastic delays.

Despite exciting new applications and major theoretical advances, it remains unclear how widely applicable nonsmooth models are, or in what sense they approximate discontinuities in real world systems. It is even unclear how to correctly simulate or solve nonsmooth systems, or how robust such solutions are to perturbation. To move closer towards these goals, here we survey one of the main approaches to modeling nonsmooth dynamics, and look at how loosening some of its rigourous but idealized framework allows us to probe its modeling assumptions. We also draw together a range of phenomena that characterize the sensitivity and robustness of nonsmooth dynamical models.