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# Modeling with nonsmooth dynamics

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# Contents

<b>1</b>	<b>Mathematics for a nonsmooth world</b> .....	1
1.1	What are discontinuities hiding? .....	3
1.2	Control switching .....	8
1.3	Sticky genes .....	11
1.4	The plan .....	13
<b>2</b>	<b>1930-2010: Nonsmooth dynamics' linear age</b> .....	15
<b>3</b>	<b>Discontinuities to model missing knowledge</b> .....	21
<b>4</b>	<b>Three experiments</b> .....	25
4.1	Filippov's convexity paradox .....	26
4.2	On or off genes .....	28
4.3	Jittery investments .....	30
<b>5</b>	<b>Layers and implementations</b> .....	35
5.1	Linear switching .....	37
5.2	Nonlinear switching .....	38
5.3	Hidden terms – the 'ghosts' of switching .....	41
<b>6</b>	<b>Ideal and non-ideal sliding</b> .....	45
6.1	Sliding perspective I: the piecewise-smooth system .....	47
6.2	Sliding perspective II: hybrid implementations .....	49
6.3	Sliding perspective III: smoothed implementations .....	52
<b>7</b>	<b>The three experiments revisited</b> .....	57
7.1	Filippov's paradox revisited .....	57
7.2	Genes revisited .....	61
7.3	Investments revisited .....	65

<b>8</b>	<b>Further curiosities of hidden dynamics</b> .....	73
8.1	The phenomenon of jitter .....	73
8.2	Hidden oscillations and chaos .....	74
<b>9</b>	<b>Closing remarks: open challenges</b> .....	79
<b>A</b>	<b>Nonsmooth models as asymptotic series</b> .....	83
A.1	The general expression .....	83
A.2	The example of the error function .....	84
<b>B</b>	<b>Simple examples of hulls, canopies, and indexing</b> .....	87
<b>C</b>	<b>Deriving the hidden terms in lemma 6.2</b> .....	89
	<b>References</b> .....	91
	<b>Index</b> .....	101

# Preface

As mathematics is applied to model ever new problems in engineering and the life sciences, increasing use is being made of systems that switch between different sets of equations on distinct domains. To find their dynamics requires the discontinuity between domains to be resolved or ‘regularized’ in some way, and there exist a range of methods to do so. Some preserve the ideal character of the discontinuity as a piecewise-smooth system (giving e.g. ‘impact’ or ‘switching’ dynamics), while others blur the discontinuity by smoothing it out, or introducing overshoots due to deterministic or stochastic delays.

Despite exciting new applications and major theoretical advances, it remains unclear how widely applicable nonsmooth models are, or in what sense they approximate discontinuities in real world systems. It is even unclear how to correctly simulate or solve nonsmooth systems, or how robust such solutions are to perturbation. To move closer towards these goals, here we survey one of the main approaches to modeling nonsmooth dynamics, and look at how loosening some of its rigorous but idealized framework allows us to probe its modeling assumptions. We also draw together a range of phenomena that characterize the sensitivity and robustness of nonsmooth dynamical models.